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TECHNICAL NOTE

D-710

POWER INPUT TO A SMALL FLAT PLATE FROM A DIFFUSELY RADIATING SPHERE, WITH APPLICATION TO EARTH SATELLITES

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SUMMARY

A general derivation is given for the radiation incident on a small flat plate from a uniformly radiating sphere. The results are presented as a function of the separation of the bodies and the orientation of the plate. The derived equations permit a determination of the total power input to a plate, whose absorptivity is to be defined, from a radiating sphere whose surface properties are to be defined. In addition, a series of curves is given which represents the power input from earth radiation to one side of a flat plate for various orientations of the plate, for a range of altitudes from 200 km to 32,000 km above the earth. These curves are based upon the assumption that the earth is a uniform diffuse emitter radiating as a black body at a temperature of 250°K. The instantaneous earth-emitted radiation incident on the elements of the surface of any satellite can be determined with these curves.

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INTRODUCTION

The impetus for consideration of this problem came from the need for a convenient method of determining the radiative power input from the earth to a given side of a flat plate oriented in an arbitrary fashion; e.g., a paddle containing solar cells affixed to an artificial satellite. Because of the fact that, at least to a first approximation, the earth can be considered a black body radiating uniformly at approximately 250°K (Reference 1), the treatment presented herein considers a diffusely emitting* sphere with a uniform emittance per unit area of the surface; and this method is equally applicable to diffusely emitted or reflected thermal radiation or to visible radiation so long as the source is uniform. This general emittance is represented by the quantity Λ. If thermal radiation is being considered it follows that $\Lambda = \sigma T^4$, where σ is the Stefan-Boltzmann constant and T is the absolute temperature for a black body.

This problem can be divided into two parts, one of which is much less difficult than the other. The first part, called Case I, arises when the plate is so oriented that the sphere, as seen from the plate, appears as a circular disk (Figure 1). However, when the plate is so oriented that its plane falls within the tangent cone (the cone defining the limits of the facing disk of the sphere), the side of the plate in question receives radiation from only a portion of the disk. In other words, self-shielding of the plate destroys the ease of integration of Case I and introduces complicated integrals (Case II) over the visible portions of the disk (Figures 2 and 3).

*A diffuse emitter is one for which the emitted power in any direction varies as the cosine of the angle between the given direction and the normal to the emitting element.

CASE I: DISK FULLY VISIBLE

For any general calculation of this type, dealing only with incoherent diffuse radiation, the integration over the surface of the sphere can be replaced by an integration over the circular disk, provided that the solid angle subtended by the disk equals that subtended by the sphere (Reference 2).

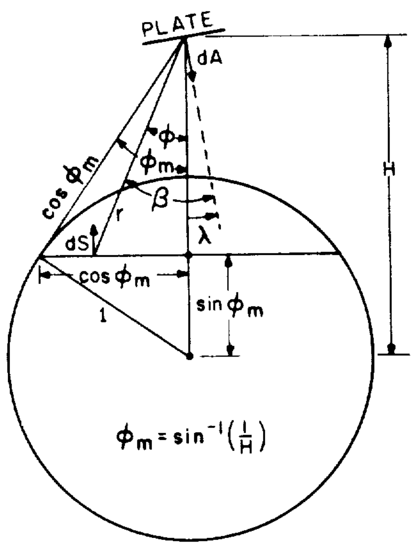

Figure 1 - Geometry of Case I

The geometry of Case I is shown in Figure 1. We shall define the following quantities:

H = the distance between the plate and the center of the sphere in units of sphere radii;

λ = the angle between the normal to the plate and the line of centers H;

ϕ_m = the maximum angle subtended by the sphere;

dA = the elemental area of the plate;

dS = the elemental area of the disk which replaces the sphere;

ϕ = the angle of integration defining the position of dS;

$d\Omega$ = the solid angle subtended at dS by the plate;

β = the angle between dA and r;

r = the radius vector between dA and dS.

Figure 1 also defines the limits of Case I:

$$\lambda + \phi_m \leq \frac{\pi}{2} .$$

We now introduce two new quantities, ϵ and α, which represent the generalized gray body emissivity of the sphere and generalized absorptivity of the plate, respectively. The term *generalized* indicates that these quantities are applicable to all possible types of radiation. The general power input to the plates can be written as

$$P_1 = \frac{\Lambda \epsilon \alpha}{\pi} \int d\Omega \, dS_\perp , \tag{1}$$

where

$$d\Omega = \frac{dA \cos \beta}{r^2}$$

and $dS_\perp$ is the component of dS perpendicular to r.

The solid angle subtended by the plate at each element $dS_\perp$ varies as we traverse the circle corresponding to constant ϕ, because $\cos\beta$ varies with azimuthal position. Consequently we can utilize the average value of $\cos\beta$ for the complete circle,

$$\overline{\cos\beta} = \cos\lambda \;\; \cos\phi, \tag{2}$$

where the $\cos\lambda$ term gives the projection of dA along H; it will be seen later that this simplification is not possible for Case II. Also, since we have taken an azimuthal average of the solid angle $d\Omega$ we can take for $dS_\perp$ the total area of an elemental ring given by

$$\overline{dS_\perp} = 2\pi r^2 \sin\phi \, d\phi . \tag{3}$$

Equation 1 becomes

$$P_1 = 2\Lambda\epsilon\alpha \, dA \cos\lambda \int_0^{\phi_m} \sin\phi \cos\phi \, d\phi . \tag{4}$$

Carrying out the integration, we have

$$P_1 = 2\Lambda\epsilon\alpha \, dA \cos\lambda \left[\frac{\sin^2\phi}{2}\right]_0^{\phi_m} = \Lambda\epsilon\alpha \, dA \cos\lambda \, \sin^2\phi_m ; \tag{5}$$

but $\sin\phi_m = 1/H$ (cf. Figure 1), and therefore

$$P_1 = \frac{\Lambda\epsilon\alpha \, dA \cos\lambda}{H^2} . \tag{6}$$

CASE II: DISK PARTLY VISIBLE

Part A: $\pi/2 - \phi_m < \lambda \leq \pi/2$

The geometry for Case II is defined in Figure 2. Both Figures 2 and 3 show the limits of the visible portion of the disk. Careful examination of Figure 3 shows that two subregions, IIA_1, and IIA_2, must be considered:

(IIA_1) $\quad 0 < \phi \leq \pi/2 - \lambda ;$

(IIA_2) $\quad \pi/2 - \lambda < \phi < \phi_m .$

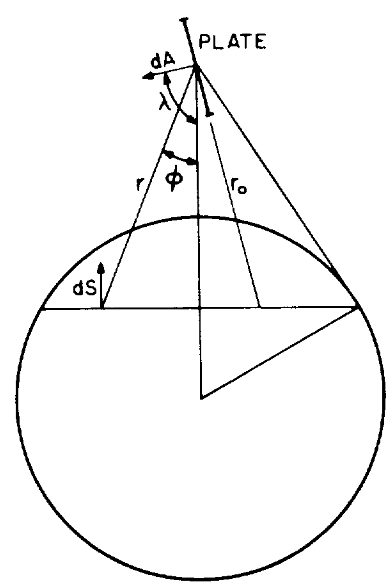

Figure 2 - Geometry of Case II, Part A

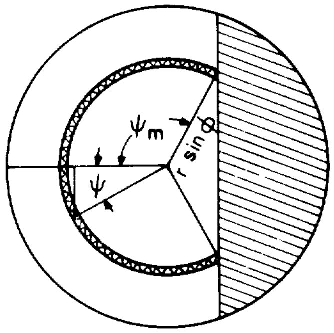

Figure 3 - Limits of integration of ψ for Case II, Part A

Figure 3 shows the elemental ring given by ϕ = constant, over which we shall integrate. It is necessary at this stage to introduce an azimuthal angle ψ if the correct value of dS is to be determined. As shown, ψ has the range $0 \le \psi \le \psi_m$ where, in general, $\psi_m = \psi(\lambda, \phi)$. When $\phi \le \pi/2 - \lambda$ we are in subregion IIA_1 where $\psi_m = \pi$; but when $\phi > \pi/2 - \lambda$, we are in subregion IIA_2, and ψ_m lies somewhere in the region $\pi/2 \le \psi_m < \pi$. In addition to the quantity ψ_m in subregion IIA_2, the angle β introduced previously must be calculated. In general, $\beta = \beta(\lambda, \phi, \psi)$.

The general expression for the power input to the plate (for Case II, Part A) is

$$P_{2A} = \frac{2\Lambda\epsilon\alpha\ dA}{\pi} \int_0^{\pi/2-\lambda} \sin\phi\ d\phi \int_0^{\pi} \cos\beta\ d\psi + \frac{2\Lambda\epsilon\alpha\ dA}{\pi} \int_{\pi/2-\lambda}^{\phi_m} \sin\phi\ d\phi \int_0^{\psi_m} \cos\beta\ d\psi. \tag{7}$$

Now $\psi_m = \psi(\phi, \lambda)$ can be determined in subregion IIA_2 by referring to Figures 1, 2, and 3. We define a quantity r_0 which equals r when $\phi = \pi/2 - \lambda$. Then

$$r \sin\phi \cos(\pi - \psi_m) = r_0 \sin(\pi/2 - \lambda). \tag{8}$$

Expanding the cosine and sine terms we have

$$r \sin\phi \cos\psi_m = -r_0 \cos\lambda. \tag{9}$$

From Figure 1,

$$r_0 \cos(\pi/2 - \lambda) = H - \sin\phi_m; \tag{10}$$

then

$$r_0 = \frac{H - \sin\phi_m}{\sin\lambda}, \qquad \text{and} \qquad r = \frac{H - \sin\phi_m}{\cos\phi}. \tag{11}$$

Substituting Equation 11 into Equation 9 yields

$$\cos\psi_m = -\frac{\cos\lambda \cos\phi}{\sin\lambda \sin\phi}. \tag{12}$$

To determine $\beta = \beta(\lambda, \phi, \psi)$, consider Figure 4. Projecting the radius r onto the plane of the plate defines the shaded triangle ABQ, from which $\cos\beta$ can be determined.

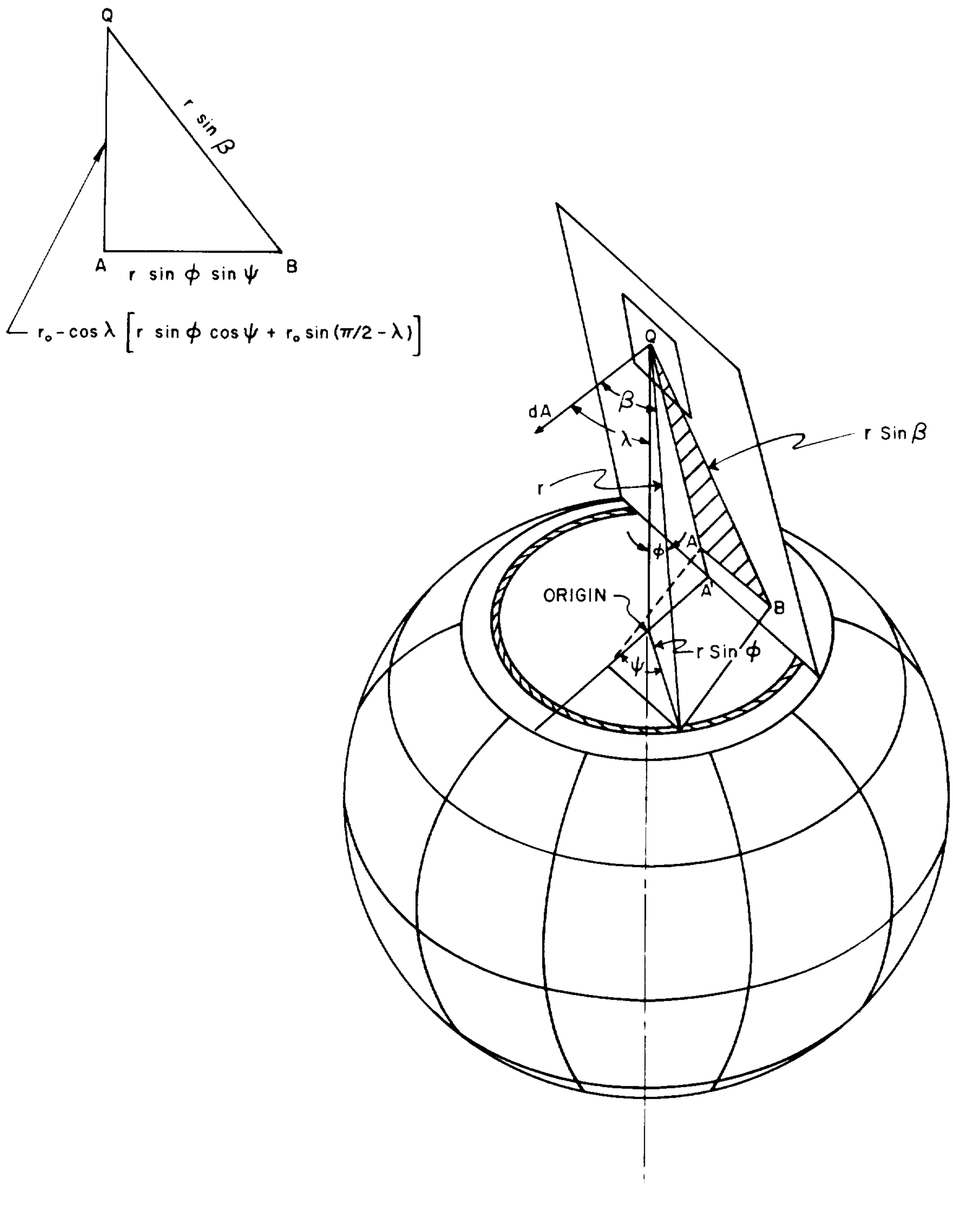

Figure 4 - Geometric construction used to determine the angle β

From Figure 4, it is seen that

$$\sin^2\beta = \frac{r_0^2 \sin^4\lambda}{r^2} - \frac{2r_0}{r} \sin^2\lambda \cos\lambda \sin\phi \cos\psi + \sin^2\phi \sin^2\psi + \cos^2\lambda \sin^2\phi \cos^2\psi. \quad (13)$$

With the value of r_0 from Equation 11, Equation 13 becomes

$$\sin^2\beta = 1 - \cos^2\beta = \sin^2\lambda \cos^2\phi - 2 \sin\lambda \cos\lambda \sin\phi \cos\phi \cos\psi + \cos^2\lambda \sin^2\phi \cos^2\psi + \sin^2\phi \sin^2\psi . \quad (14)$$

Therefore

$$\cos^2\beta = 1 - \sin^2\lambda \cos^2\phi + 2 \sin\lambda \cos\lambda \sin\phi \cos\phi \cos\psi - \sin^2\phi \sin^2\psi - \cos^2\lambda \sin^2\phi \cos^2\psi \quad (15)$$

which becomes, with suitable trigonometric manipulation,

$$\cos^2\beta = \sin^2\phi \sin^2\lambda \cos^2\psi + 2 \sin\lambda \cos\lambda \sin\phi \cos\phi \cos\psi + \cos^2\phi \cos^2\lambda . \quad (16)$$

Since the right-hand member of Equation 16 is a perfect square,

$$\cos\beta = \cos\lambda \cos\phi + \sin\lambda \sin\phi \cos\psi . \quad (17)$$

Now, with the results of Equation 17, Equation 7 becomes

$$P_{2A} = \frac{2\Lambda\epsilon\alpha\ dA}{\pi} \int_0^{\pi/2-\lambda} \sin\phi\, d\phi \int_0^{\pi} (\cos\lambda \cos\phi + \sin\lambda \sin\phi \cos\psi)\, d\psi + \frac{2\Lambda\epsilon\alpha\ dA}{\pi} \int_{\pi/2-\lambda}^{\phi_m} \sin\phi\, d\phi \int_0^{\psi_m} (\cos\lambda \cos\phi + \sin\lambda \sin\phi \cos\psi)\, d\psi . \quad (18)$$

We shall first integrate over ψ. The first of the two ψ integrals is

$$\int_0^{\pi} (\cos\lambda \cos\phi + \sin\lambda \sin\phi \cos\psi)\, d\psi = \pi \cos\lambda \cos\phi ; \quad (19)$$

and the second integral is

$$\int_0^{\psi_m} (\cos\lambda \cos\phi + \sin\lambda \sin\phi \cos\psi)\, d\psi = \cos\lambda \cos\phi \cos^{-1}\left(- \frac{\cos\lambda \cos\phi}{\sin\lambda \sin\phi}\right) + (\sin^2\phi - \cos^2\lambda)^{\frac{1}{2}} , \quad (20)$$

where, Equation 12 is used for the value of ψ_m.

Equation 18 now becomes

$$P_{2A} = 2\Lambda\epsilon\alpha\, dA \int_0^{\pi/2-\lambda} \cos\lambda \sin\phi \cos\phi\, d\phi + \frac{2\Lambda\epsilon\alpha\, dA}{\pi} \int_{\pi/2-\lambda}^{\sin^{-1} 1/H} \sin\phi (\sin^2\phi - \cos^2\lambda)^{\frac{1}{2}}\, d\phi$$

$$+ \frac{2\Lambda\epsilon\alpha\, dA}{\pi} \int_{\pi/2-\lambda}^{\sin^{-1} 1/H} \cos\lambda \sin\phi \cos\phi \cos^{-1}\left(-\frac{\cos\lambda\cos\phi}{\sin\lambda\sin\phi}\right) d\phi\,. \tag{21}$$

We next evaluate the three integrals. The first one can be done directly:

$$2\Lambda\epsilon\alpha\, dA \cos\lambda \int_0^{\pi/2-\lambda} \sin\phi\cos\phi\, d\phi = \Lambda\epsilon\alpha\, dA \cos^3\lambda\,. \tag{22}$$

The second integral is

$$\frac{2\Lambda\epsilon\alpha\, dA}{\pi} \int_{\pi/2-\lambda}^{\sin^{-1} 1/H} \sin\phi(\sin^2\phi - \cos^2\lambda)^{\frac{1}{2}}\, d\phi = -\frac{2\Lambda\epsilon\alpha\, dA}{\pi} \int_{\sin\lambda}^{\sqrt{1-1/H^2}} (\sin^2\lambda - z^2)^{\frac{1}{2}}\, dz\,, \tag{23}$$

with the change of variable

$$\cos\phi = z \qquad \text{and} \qquad dz = -\sin\phi\, d\phi\,; \tag{24}$$

thus the second integral of Equation 21 becomes

$$-\frac{2\Lambda\epsilon\alpha\, dA}{\pi}\left\{\frac{z(\sin^2\lambda - z^2)^{\frac{1}{2}}}{2} + \frac{\sin^2\lambda}{2}\sin^{-1}\left[\frac{z}{\sin\lambda}\right]\right\}_{\sin\lambda}^{\sqrt{1-1/H^2}} \tag{25}$$

which, upon insertion of the limits, is

$$-\frac{\Lambda\epsilon\alpha\, dA}{\pi}\left\{\left(1-\frac{1}{H^2}\right)^{\frac{1}{2}}\left[\sin^2\lambda - \left(1-\frac{1}{H^2}\right)\right]^{\frac{1}{2}} + \sin^2\lambda\sin^{-1}\left[\frac{(1-1/H^2)^{\frac{1}{2}}}{\sin\lambda}\right] - \frac{\pi\sin^2\lambda}{2}\right\}. \tag{26}$$

By collecting terms, we can now rewrite Equation 23:

$$\frac{2\Lambda\epsilon\alpha\, dA}{\pi}\int_{\pi/2-\lambda}^{\sin^{-1} 1/H} \sin\phi(\sin^2\phi - \cos^2\lambda)^{\frac{1}{2}}\, d\phi$$

$$= \frac{-\Lambda\epsilon\alpha\, dA \sin^2\lambda}{\pi}\left[\frac{(H^2-1)^{\frac{1}{2}}(1-H^2\cos^2\lambda)^{\frac{1}{2}}}{H^2\sin^2\lambda} - \frac{\pi}{2} + \sin^{-1}\left(\frac{(H^2-1)^{\frac{1}{2}}}{H\sin\lambda}\right)\right]. \tag{27}$$

Now consider the third integral of Equation 21; this integral may be integrated by parts directly if the following substitutions are made:

let

$$u = \cos^{-1}(-\cot\lambda\cot\phi), \qquad du = \frac{-\csc^2\phi\,d\phi}{(\tan^2\lambda - \cot^2\phi)^{\frac{1}{2}}};$$

and

$$dv = \sin\phi\cos\phi\,d\phi, \qquad v = \frac{\sin^2\phi}{2}.$$

The partial integration then gives

$$\frac{2\Lambda\epsilon a\,dA}{\pi}\int u\,dv$$

$$= \frac{2\Lambda\epsilon a\,dA}{\pi}\left[\frac{\cos\lambda}{2}\sin^2\phi\cos^{-1}(-\cot\lambda\cot\phi) + \frac{\cos\lambda}{2}\int\frac{\sin^2\phi\csc^2\phi\,d\phi}{(\tan^2\lambda - \cot^2\phi)^{\frac{1}{2}}}\right]. \tag{28}$$

Since the sine and cosecant terms go out in the second term, we have an additional integration to perform which is (with the proper limits inserted):

$$\frac{\cos\lambda}{2}\int_{\pi/2-\lambda}^{\sin^{-1}1/H}\frac{d\phi}{(\tan^2\lambda - \cot^2\phi)^{\frac{1}{2}}} \equiv \frac{\cos^2\lambda}{2}\int_{\pi/2-\lambda}^{\sin^{-1}1/H}\frac{\sin\phi\,d\phi}{(\sin^2\lambda - \cos^2\phi)^{\frac{1}{2}}}. \tag{29}$$

If we make the change of variable $u = \cos\phi$, $du = -\sin\phi\,d\phi$, Equation 29 becomes

$$\frac{\cos^2\lambda}{2}\int_{\pi/2-\lambda}^{\sin^{-1}1/H}\frac{\sin\phi\,d\phi}{(\sin^2\lambda - \cos^2\phi)^{\frac{1}{2}}} = \frac{-\cos^2\lambda}{2}\int_{\sin\lambda}^{\sqrt{1-1/H^2}}\frac{du}{(\sin^2\lambda - u^2)^{\frac{1}{2}}}$$

$$= \frac{\cos^2\lambda}{2}\left\{\frac{\pi}{2} - \sin^{-1}\left[\frac{(H^2-1)^{\frac{1}{2}}}{H\sin\lambda}\right]\right\}. \tag{30}$$

Consequently, inserting the limits into the first term of Equation 28 and combining with Equation 30 gives the final form of the third integral of Equation 21:

$$\frac{2\Lambda\epsilon a\,dA}{\pi}\int_{\pi/2-\lambda}^{\sin^{-1}1/H}\cos\lambda\sin\phi\cos\phi\cos^{-1}\left(-\frac{\cos\lambda\cos\phi}{\sin\lambda\sin\phi}\right)d\phi$$

$$= \frac{2\Lambda\epsilon a\,dA}{\pi}\left\{\frac{\cos\lambda}{2H^2}\cos^{-1}\left(-(H^2-1)^{\frac{1}{2}}\cot\lambda\right) - \frac{\pi\cos^3\lambda}{2} + \frac{\cos^2\lambda}{2}\left[\frac{\pi}{2} - \sin^{-1}\left(\frac{(H^2-1)^{\frac{1}{2}}}{H\sin\lambda}\right)\right]\right\}. \tag{31}$$

Upon substitution of the results of Equations 22 through 31 into Equation 21 we have the final expression for the power input:

$$P_{2A} = \frac{2\Lambda\epsilon\alpha\ dA}{\pi}\left[\frac{\pi}{4} - \frac{\sin^{-1}}{2}\left[\frac{(H^2-1)^{\frac{1}{2}}}{H\sin\lambda}\right] + \frac{1}{2H^2}\left\{\cos\lambda\cos^{-1}\left[-(H^2-1)^{\frac{1}{2}}\cot\lambda\right] - (H^2-1)^{\frac{1}{2}}\left[1 - H^2\cos^2\lambda\right]^{\frac{1}{2}}\right\}\right]. \qquad (32)$$

Now, if the terms in the large brackets, multiplied by $2/\pi$, are represented by Y_1, a function of H and λ, then

$$P_{2A} = \Lambda\epsilon\alpha\ dA\ Y_1(H,\lambda)\ , \qquad (33)$$

where $Y_1(H)$ can be computed for various values of the parameter λ and plotted for convenient use.

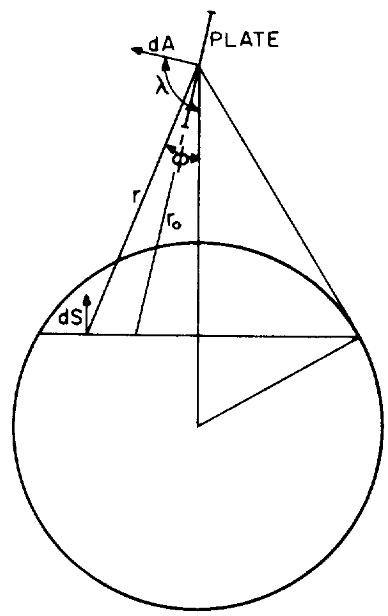

Figure 5 - Geometry of Case II, Part B

Part B: $\pi/2 < \lambda \leq \pi/2 + \phi_m$

The geometrical situation of Part B is shown in Figures 5 and 6. It is clear that this case is simpler than before because only one possible region is under consideration, where ψ_m is given by Equation 12. The range of integration of ϕ is given by $(\lambda - \pi/2) \leq \phi \leq \phi_m$. A check of the earlier expression given for $\cos\beta$ will show that it is still applicable in the present situation.

The general expression for the power input to the plate (for Case II, Part B) can be written:

$$P_{2B} = \frac{2\Lambda\epsilon\alpha\ dA}{\pi}\int_{\lambda-\pi/2}^{\phi_m}\int_0^{\psi_m}\cos\beta\sin\phi\ d\phi\ d\psi\ , \qquad (34)$$

which is nothing more than Equation 7 with new limits on the ϕ integration. In expanded form this equation is given by the second part of Equation 18.

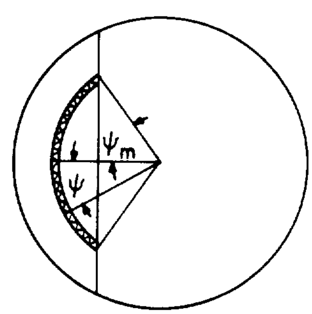

Figure 6 - Limits of integration of ψ for Case II, Part B

When the integration over ψ is carried out, the last two terms of Equation 21 remain with a suitable change in limits:

$$P_{2B} = \frac{2\Lambda\epsilon\alpha\ dA}{\pi} \int_{\lambda-\pi/2}^{\sin^{-1} 1/H} \sin\phi(\sin^2\phi - \cos^2\lambda)^{\frac{1}{2}}\, d\phi$$

$$+ \frac{2\Lambda\epsilon\alpha\ dA}{\pi} \int_{\lambda-\pi/2}^{\sin^{-1} 1/H} \cos\lambda \sin\phi \cos\phi \cos^{-1}\left(- \frac{\cos\lambda \cos\phi}{\sin\lambda \sin\phi}\right) d\phi\ . \qquad (35)$$

The first integral in Equation 35 is identical to the one integrated in Equations 23 through 27, while the second integral is identical in form to the one integrated in Equations 28 through 31. The only difference in the result is that the arc-cosine terms are not equal because the values of λ are not the same, and in the integration by parts (Equation 28) the du has changed sign. However, so long as we remain cognizant of the limits of λ in each case no trouble will arise. Consequently, the result can be written:

$$P_{2B} = \frac{2\Lambda\epsilon\alpha\ dA}{\pi} \left[\frac{\pi}{4} - \frac{\sin^{-1}}{2} \left[\frac{(H^2-1)^{\frac{1}{2}}}{H \sin\lambda}\right] \right.$$

$$\left. + \frac{1}{2H^2} \left\{ \cos\lambda \cos^{-1} \left[- (H^2-1)^{\frac{1}{2}} \cot\lambda \right] - (H^2-1)^{\frac{1}{2}} \left[1 - H^2 \cos^2\lambda\right]^{\frac{1}{2}} \right\} \right] . \qquad (36)$$

Now we let the expression in this set of large brackets, multiplied by $2/\pi$, equal the function Y_2, and thus

$$P_{2B} = \Lambda\epsilon\alpha\ dA\ Y_2(H,\lambda)\ , \qquad (37)$$

where $Y_2(H)$ can be computed for various values of λ and plotted for convenient use.

RESULTS

The geometrical dependence (i.e., the functions Y_1 and Y_2) of the radiation calculation for Case II, Parts A and B (sometimes called the shape modulus) is presented graphically in Figures 7 and 8. The input to the plate for any arbitrary orientation can immediately be determined by using Figures 7 and 8, in conjunction with Equations 6, 33, and 37.

Up to this point, the input in question concerns only one side of the plate — called side A. The input to the other side — called B, which in the figures would be the side whose normal points up — clearly follows from the observation that the situation, from one side to the other, is symmetric about $\lambda = \pi/2$. In other words, the input to side B at $\lambda = \pi/2 - x$ degrees is equal to the input to side A at $\lambda = \pi/2 + x$ degrees, and vice-versa. Thus, to determine the input to side B when the normal to side A makes an angle λ with the line of centers, merely replace λ by $\pi - \lambda$.

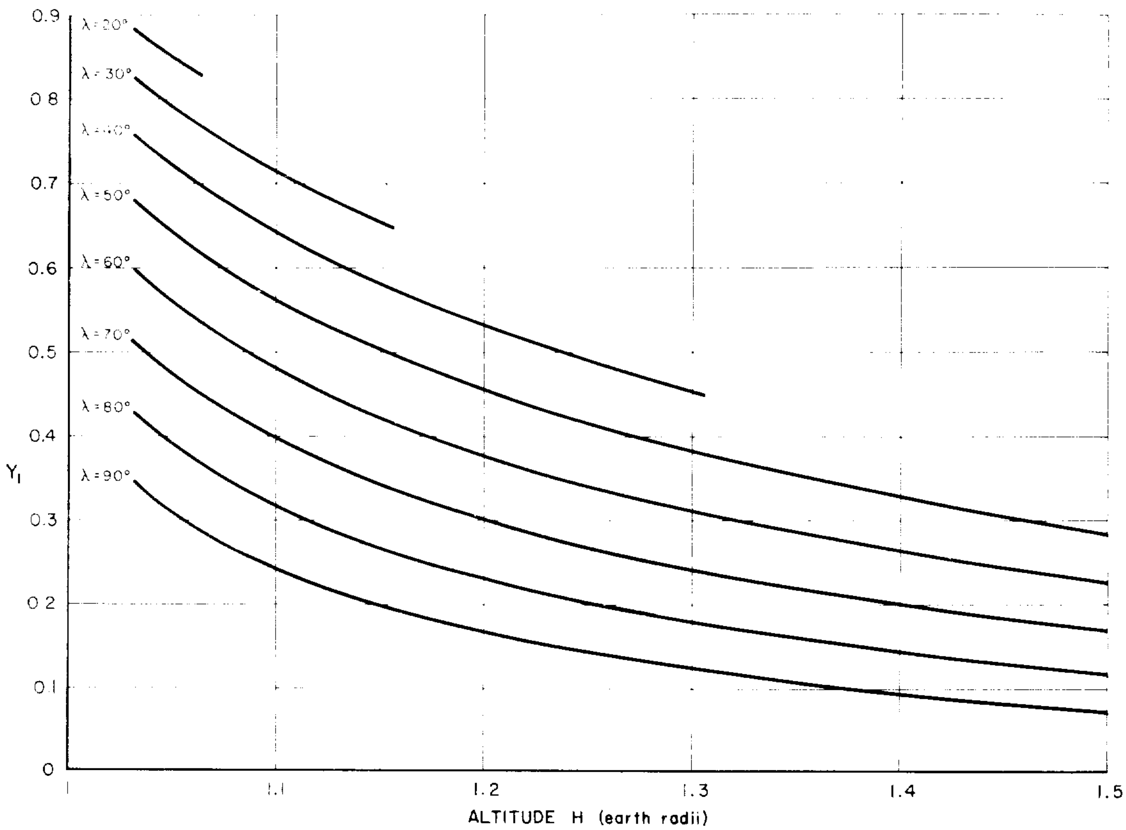

Figure 7 - Geometrical dependence of the radiation calculation (shape modulus) for Case II, Part A

As a direct application of the preceding calculations a series of curves may be plotted (Figures 9 and 10) which give the earth-radiation power input to a flat plate of unit area and absorptivity 1. As a first approximation the earth can be considered to radiate as a black body at 250°K, for which $\Lambda = \sigma T^4$ is the generalized emittance. In addition, the earth is assumed to be perfectly spherical and the independent variable H is given in kilometers above the surface of the earth.

For this application Equations 6, 33, and 37, become:

$$P_1 = \frac{(2.215 \times 10^{-2}) \cos \lambda}{H^2} \text{ watts} ; \tag{38}$$

$$P_{2A} = (2.215 \times 10^{-2}) \, Y_1 \text{ watts} ; \tag{39}$$

and

$$P_{2B} = (2.215 \times 10^{-2}) \, Y_2 \text{ watts} . \tag{40}$$

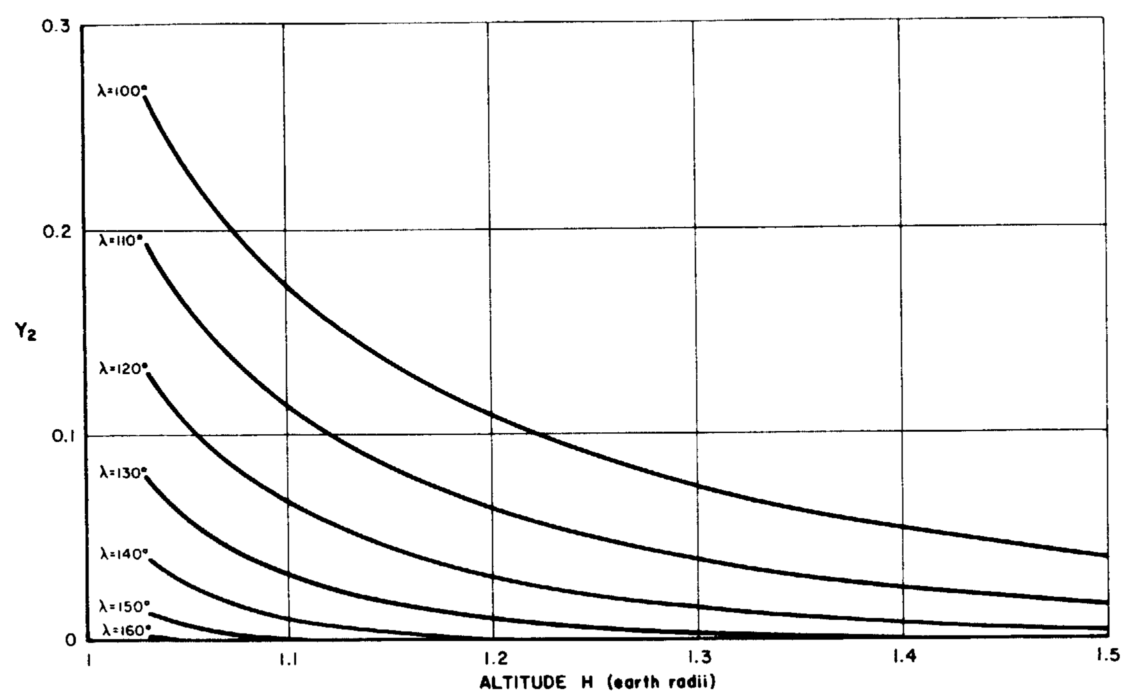

Figure 8 - Geometrical dependence of the radiation calculation (shape modulus) for Case II, Part B

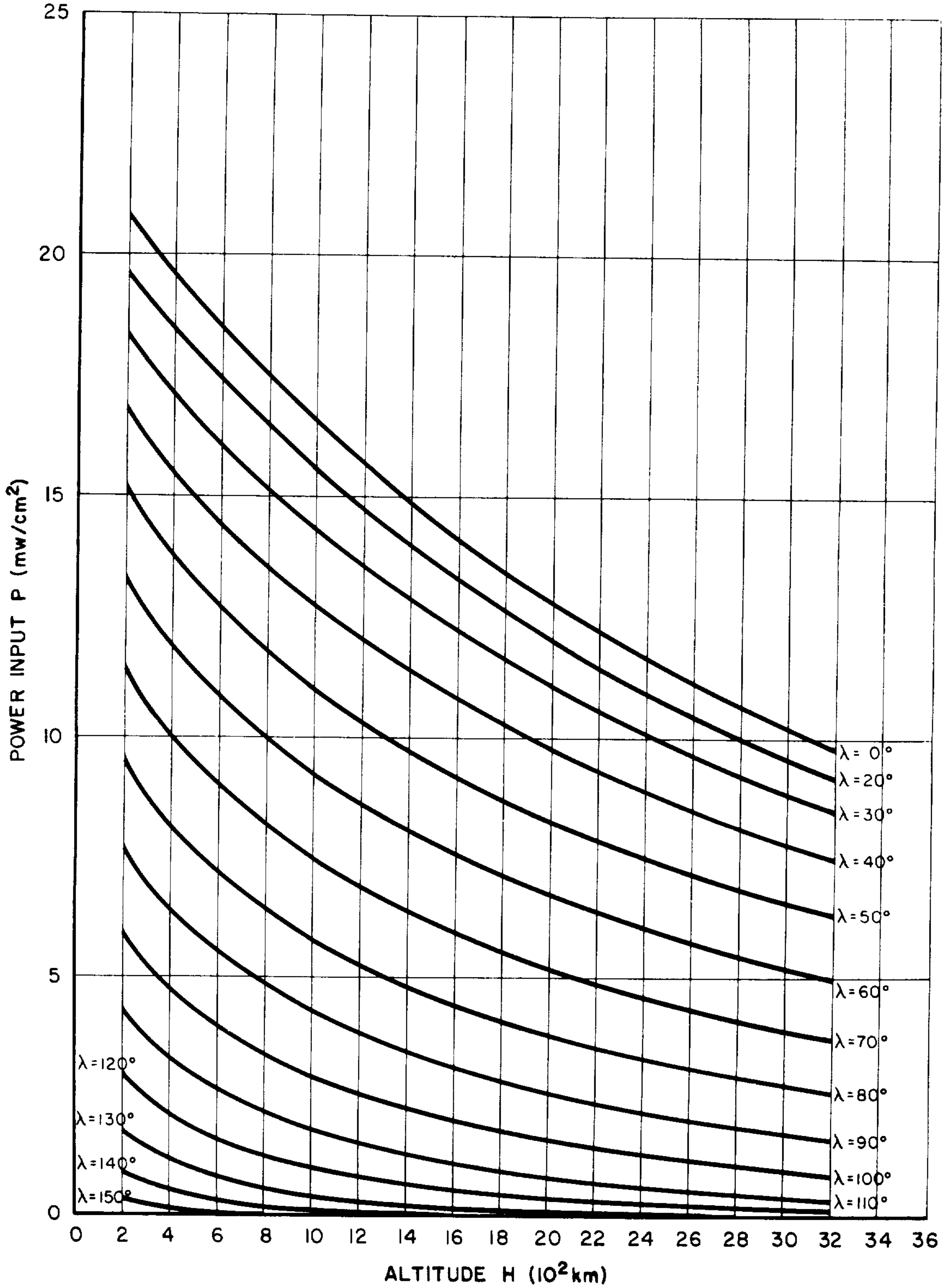

Figure 9 - Power input of thermal radiation emanating from the earth to a flat plate of absorptivity 1 where $200 \leq H \leq 3200$ km

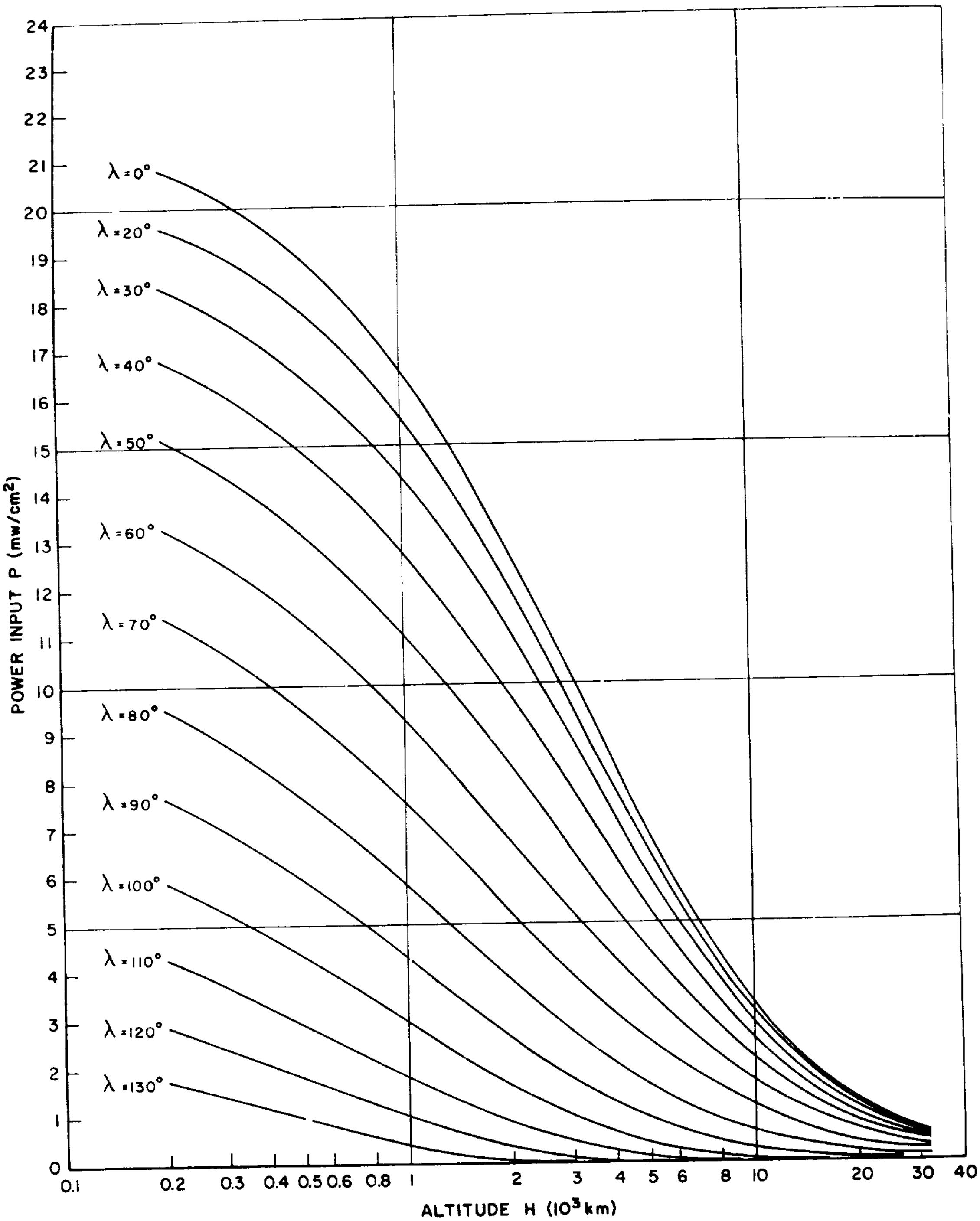

Figure 10 - Power input of thermal radiation emanating from the earth to a flat plate of absorptivity 1 for the altitude range 200 $\leq$ H $\leq$ 32,000 km

ACKNOWLEDGMENTS

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(NASA TECHNICAL NOTE D-710) CORRECTED COPY

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uniform diffuse emitter radiating as a black body at a temperature of 250° K. The instantaneous earth-emitted radiation incident on the elements of the surface of any satellite can be determined with these curves.

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